Closing tonight (11pm): 10.1
Closing Fri: 2.1, 2.2, 2.3

Entry Task: Draw rough sketches of

1. $h(x) = \begin{cases} x^2 & \text{, if } x < 0; \\ 3 & \text{, if } x = 0; \\ x & \text{, if } x > 0. \end{cases}$

2. $g(x) = \frac{1}{x^2}$

3. $j(x) = \frac{x^2-9}{x-3}$

4. $f(x) = \frac{|x|}{x}$

2.2 Limits

$$\lim_{x \to a} f(x) = L$$

"the **limit** of f(x), as x approaches a, is L". It means

as *x* takes on values closer and closer to *a*,

y = f(x) takes on values closer and closer to L.

Find

$h(0) =$

$\lim_{x \to 0} h(x) =$

$f(0) =$

$\lim_{x \to 0} f(x) =$

$g(0) =$

$\lim_{x \to 0} g(x) =$

$\lim_{x \to \infty} g(x) =$

$\lim_{x \to -\infty} g(x) =$

$j(3) =$

$\lim_{x \to 3} j(x) =$

One-sided limits

$$\lim_{x \to a^-} f(x) = L$$

"the limit of *f(x)*, as *x* approaches *a* **from the left**, is *L*". It means

as *x* takes on values closer to and **from the left** (smaller values) of *a*, *y = f(x)* takes on values closer and closer to L.

$$\lim_{x \to a^+} f(x) = L$$

"the limit of *f(x)*, as *x* approaches *a* **from the right**, is *L*".

Note:

$$\lim_{x \to a} f(x) = L \quad \text{if and only if} \quad \text{both} \begin{cases} \lim_{x \to a^-} f(x) = L \\ \lim_{x \to a^+} f(x) = L \end{cases}$$

Find

$$\lim_{x \to 0^-} f(x) =$$

$$\lim_{x \to 0^+} f(x) =$$

What if we can't easily graph?
We can try plugging in points (but be careful).

Example: Find

$$\lim_{x \to 0} \frac{\sin(x)}{x} =$$

Example: Find

$$\lim_{x \to 0} \sin\left(\frac{\pi}{x}\right) =$$

2.3 Limit Laws and Strategies

Some Basic Limit Laws:

1. $\lim_{x\to a} c = c$

2. $\lim_{x\to a} x = a$

3. $\lim_{x\to a}[f(x) + g(x)]$
$= \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$

4. $\lim_{x\to a}[f(x)g(x)]$
$= \lim_{x\to a} f(x) \lim_{x\to a} g(x)$

5. If $\boldsymbol{\lim_{x\to a} g(x) \neq 0}$, then
$$\lim_{x\to a}\left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$$

Examples:

1. $\lim_{x\to -7} 10 =$

2. $\lim_{x\to 14} x =$

3. $\lim_{x\to -2}[x + 6] = \lim_{x\to -2} x + \lim_{x\to -2} 6$

4. $\lim_{x\to 5}[2x^2] = \lim_{x\to 5} 2 \lim_{x\to 5} x \lim_{x\to 5} x$

5. $\lim_{x\to 4}\left[\frac{x+2}{x^2}\right] = \frac{\lim_{x\to 4}(x+2)}{\lim_{x\to 4} x^2}$

Limit Flow Chart for

$$\lim_{x \to a} \left[\frac{f(x)}{g(x)} \right]$$

1. Try plugging in the value.
   If denominator ≠ 0, done!

2. **If denom = 0 & numerator ≠ 0**,
   the answer is -∞, +∞ or DNE.
   Examine the sign (pos/neg) of the output from each side.

3. **If denom = 0 & numerator = 0**,
   Use algebraic methods to simplify and cancel until one of them is not zero.

Examples:

$$1. \lim_{x \to 1} \frac{x+6}{x-4} =$$

$$2a. \lim_{x \to 2^+} \frac{x+4}{x-2} =$$

$$2b. \lim_{x \to 2^-} \frac{x+4}{x-2} =$$

$$2c. \lim_{x \to 2} \frac{x+4}{x-2} =$$

$$2d. \lim_{x \to 0} \frac{\cos(x) + e^x}{x^2} =$$

For the den = 0, num = 0 case, here is a summary of some algebra to try:

Strategy 1: Factor/Cancel
Strategy 2: Simplify Fractions
Strategy 3: Expand/Simplify
Strategy 4: Multiply by Conjugate

Strategy 5: Change Variable
Strategy 6: Compare to other functions (Squeeze Thm)

Examples:

1. $\lim_{x\to 5} \frac{x^2 - 25}{x - 5} =$

2. $\lim_{h\to 0} \frac{\frac{1}{2+h} - \frac{1}{2}}{h} =$

3. $\lim_{h \to 0} \frac{(3+h)^2 - 9}{h} =$

4. $\lim_{x \to 4} \frac{x - 4}{\sqrt{x} - 2} =$

Squeeze Thm:

If the following hold:

(1) $g(x) \leq f(x) \leq h(x)$ near $x = a$

(2) $\lim_{x \to a} g(x) = L$ and $\lim_{x \to a} h(x) = L$

then

$$\lim_{x \to a} f(x) = L$$

Example: Find

$$\lim_{x \to 0} x^2 \cos\left(\frac{10}{x}\right) =$$